Parametric Curves

A more general representation of a curve involves expressing its coordinates in terms of another independent variable or parameter as follows

$$x = f(u), y = g(u), u \in [a, b]$$

Each value of u corresponds to a coordinate pair (f(u), g(u)). If we collect all the points defined by u in a specific interval, we get a parametric curve. For example, the definition

$$x = \cos u, y = \sin u, u \in [0, 2\pi]$$

defines a unit circle, centered at the origin, which begins and ends at (1,0).

In order to visualize a parametric curve in Matlab, we first define a set of points on the u-domain.

```
u = linspace(0,2*pi,100)
```

```
u = 1×100
         0    0.0635    0.1269    0.1904    0.2539    0.3173    0.3808    0.4443 ···
```

Next we define a function that accepts as u as input, and returns the coordinates x and y as output. The function is defined at the end of this script, but we include it here for clarity

function [x,y] = circle(u)

x = cos(u)

y = sin(u)

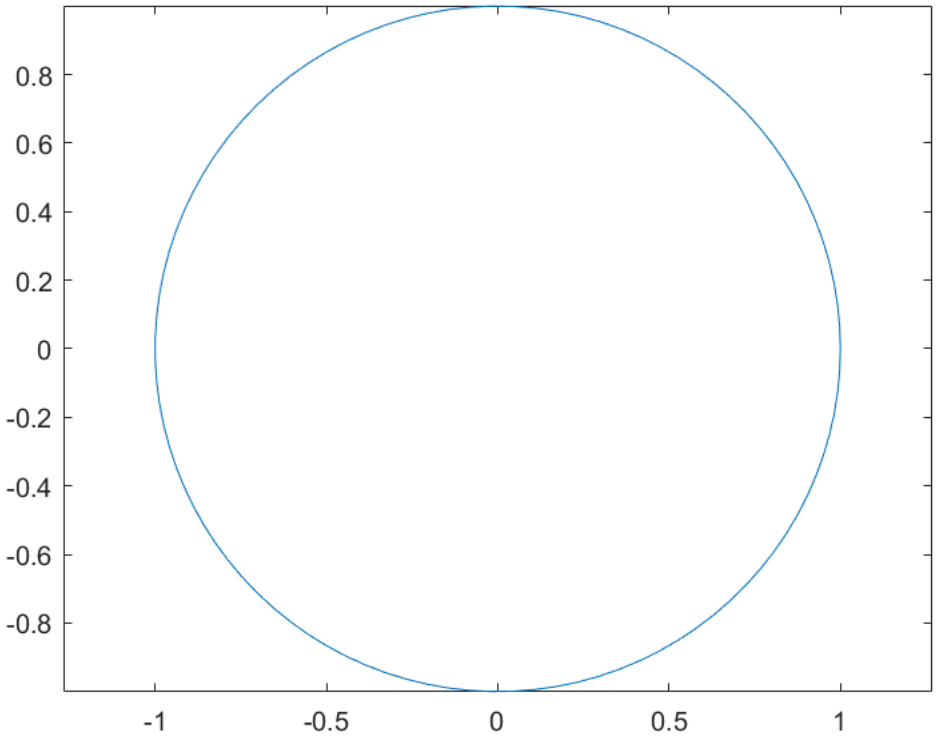
end

Now with the function defined, we can call it and then use **plot** as before, setting the axes so that it is clear that we have a circle.

```
[x,y] = circle(u)
```

```
x = 1×100
    1.0000    0.9980    0.9920    0.9819    0.9679    0.9501    0.9284    0.9029 ···
y = 1×100
         0    0.0634    0.1266    0.1893    0.2511    0.3120    0.3717    0.4298 ···
```

```
plot(x,y)
axis equal
```



As u increases from 0 to 2π, the circle is traced out in counter-clockwise direction. The variable u can be interpreted as the angle from the x-axis to the current point on the circle.

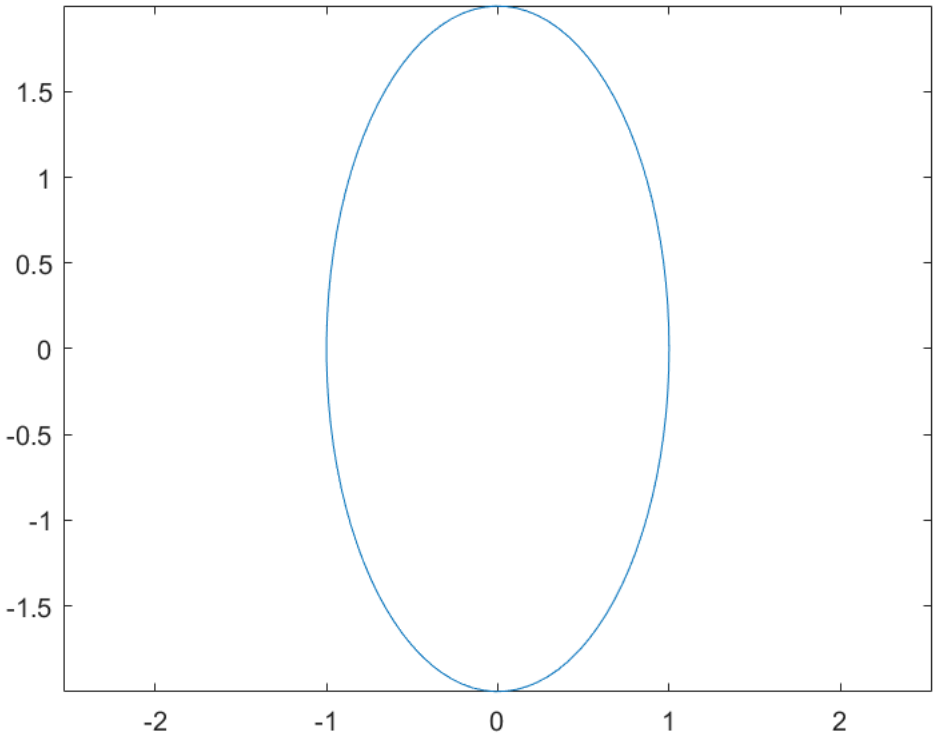
Exercise 1: Look up the parametric equations that define an ellipse, and use the techniques developed here to visualize them.

Solution: A quick browse of the internet shows that the parametric equations for an ellipse are

$$x = a\cos u, y = b\sin u, u \in [0, 2\pi]$$

where a and b are parameters. Let's visualize the ellipse corresponding to a = 1 and b = 2

```
u=linspace(0,2*pi,100);
x=1*cos(u);
y=2*sin(u);
plot(x,y)
axis equal;
```

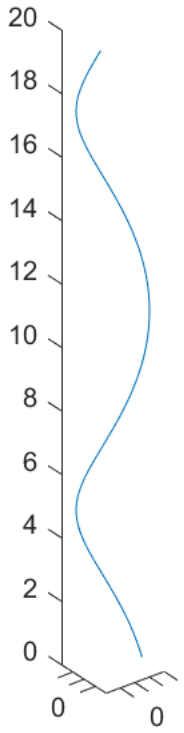


Exercise 2: A helix in 3D is often defined using the parametric equations

$$x = a\cos u, y = a\sin u, z = bu$$

where both a and b are positive. Use **plot3** to visualize the helix with a = 1 and b = 2 over the domain $u \in [0, 10]$.

```
u=linspace(0,10,500);
x=1*cos(u);
y=1*sin(u);
z=2*u;
plot3(x,y,z)
axis equal;
```



Function definitions

```
function [x,y] = circle(u)
    % circle   Computes the coordinates of a parametric circle
    x = cos(u);
    y = sin(u);
end
```