

Surfaces defined implicitly by functions

A surface in three dimensions can also be implicitly defined by a function of three variables, $f(x, y, z) = 0$. For example, the set of points (x, y, z) that satisfy the following equation

$$x^2 + y^2 + z^2 - 1 = 0$$

defines a sphere of radius 1, centered at the origin. A surface defined implicitly can be visualized using **isosurface** in Matlab. First, we will define a mesh in 3D

```
x = linspace(-2,2,50)

x = 1x50
-2.0000 -1.9184 -1.8367 -1.7551 -1.6735 -1.5918 -1.5102 -1.4286 ...

y = linspace(-2,2,50)

y = 1x50
-2.0000 -1.9184 -1.8367 -1.7551 -1.6735 -1.5918 -1.5102 -1.4286 ...

z = linspace(-2,2,50)

z = 1x50
-2.0000 -1.9184 -1.8367 -1.7551 -1.6735 -1.5918 -1.5102 -1.4286 ...

[X,Y,Z] = meshgrid(x,y,z)

X =
X(:, :, 1) =

-2.0000 -1.9184 -1.8367 -1.7551 -1.6735 -1.5918 -1.5102 -1.4286 -1.3469 -1.2653 -1.1837 -1.1020 -1.0203 -0.9386 -0.8569 -0.7752 -0.6935 -0.6118 -0.5301 -0.4484 -0.3667 -0.2850 -0.2033 -0.1216 -0.0399 0.0418 0.1235 0.2052 0.2869 0.3686 0.4503 0.5320 0.6137 0.6954 0.7771 0.8588 0.9405 1.0222 1.1039 1.1856 1.2673 1.3490 1.4307 1.5124 1.5941 1.6758 1.7575 1.8392 1.9209 2.0026 2.0843 2.1660 2.2477 2.3294 2.4111 2.4928 2.5745 2.6562 2.7379 2.8196 2.9013 2.9830 3.0647 3.1464 3.2281 3.3098 3.3915 3.4732 3.5549 3.6366 3.7183 3.8000 3.8817 3.9634 4.0451 4.1268 4.2085 4.2902 4.3719 4.4536 4.5353 4.6170 4.6987 4.7804 4.8621 4.9438 5.0255 5.1072 5.1889 5.2706 5.3523 5.4340 5.5157 5.5974 5.6791 5.7608 5.8425 5.9242 6.0059 6.0876 6.1693 6.2510 6.3327 6.4144 6.4961 6.5778 6.6595 6.7412 6.8229 6.9046 6.9863 7.0680 7.1497 7.2314 7.3131 7.3948 7.4765 7.5582 7.6399 7.7216 7.8033 7.8850 7.9667 8.0484 8.1301 8.2118 8.2935 8.3752 8.4569 8.5386 8.6203 8.7020 8.7837 8.8654 8.9471 9.0288 9.1105 9.1922 9.2739 9.3556 9.4373 9.5190 9.6007 9.6824 9.7641 9.8458 9.9275 10.0092 10.0909 10.1726 10.2543 10.3360 10.4177 10.4994 10.5811 10.6628 10.7445 10.8262 10.9079 10.9896 11.0713 11.1530 11.2347 11.3164 11.3981 11.4798 11.5615 11.6432 11.7249 11.8066 11.8883 11.9700 12.0517 12.1334 12.2151 12.2968 12.3785 12.4602 12.5419 12.6236 12.7053 12.7870 12.8687 12.9504 13.0321 13.1138 13.1955 13.2772 13.3589 13.4406 13.5223 13.6040 13.6857 13.7674 13.8491 13.9308 14.0125 14.0942 14.1759 14.2576 14.3393 14.4210 14.5027 14.5844 14.6661 14.7478 14.8295 14.9112 14.9929 15.0746 15.1563 15.2380 15.3197 15.4014 15.4831 15.5648 15.6465 15.7282 15.8099 15.8916 15.9733 16.0550 16.1367 16.2184 16.3001 16.3818 16.4635 16.5452 16.6269 16.7086 16.7903 16.8720 16.9537 17.0354 17.1171 17.1988 17.2805 17.3622 17.4439 17.5256 17.6073 17.6890 17.7707 17.8524 17.9341 18.0158 18.0975 18.1792 18.2609 18.3426 18.4243 18.5060 18.5877 18.6694 18.7511 18.8328 18.9145 18.9962 19.0779 19.1596 19.2413 19.3230 19.4047 19.4864 19.5681 19.6498 19.7315 19.8132 19.8949 19.9766 20.0583 20.1400 20.2217 20.3034 20.3851 20.4668 20.5485 20.6302 20.7119 20.7936 20.8753 20.9570 21.0387 21.1204 21.2021 21.2838 21.3655 21.4472 21.5289 21.6106 21.6923 21.7740 21.8557 21.9374 22.0191 22.1008 22.1825 22.2642 22.3459 22.4276 22.5093 22.5910 22.6727 22.7544 22.8361 22.9178 23.0000 23.0821 23.1642 23.2463 23.3284 23.4105 23.4926 23.5747 23.6568 23.7389 23.8210 23.9031 23.9852 24.0673 24.1494 24.2315 24.3136 24.3957 24.4778 24.5599 24.6420 24.7241 24.8062 24.8883 24.9704 25.0525 25.1346 25.2167 25.2988 25.3809 25.4630 25.5451 25.6272 25.7093 25.7914 25.8735 25.9556 26.0377 26.1198 26.2019 26.2840 26.3661 26.4482 26.5303 26.6124 26.6945 26.7766 26.8587 26.9408 27.0229 27.1050 27.1871 27.2692 27.3513 27.4334 27.5155 27.5976 27.6797 27.7618 27.8439 27.9260 28.0081 28.0902 28.1723 28.2544 28.3365 28.4186 28.5007 28.5828 28.6649 28.7470 28.8291 28.9112 28.9933 29.0754 29.1575 29.2396 29.3217 29.4038 29.4859 29.5680 29.6501 29.7322 29.8143 29.8964 29.9785 30.0606 30.1427 30.2248 30.3069 30.3890 30.4711 30.5532 30.6353 30.7174 30.7995 30.8816 30.9637 31.0458 31.1279 31.2100 31.2921 31.3742 31.4563 31.5384 31.6205 31.7026 31.7847 31.8668 31.9489 32.0310 32.1131 32.1952 32.2773 32.3594 32.4415 32.5236 32.6057 32.6878 32.7699 32.8520 32.9341 33.0162 33.0983 33.1804 33.2625 33.3446 33.4267 33.5088 33.5909 33.6730 33.7551 33.8372 33.9193 34.0014 34.0835 34.1656 34.2477 34.3298 34.4119 34.4940 34.5761 34.6582 34.7403 34.8224 34.9045 34.9866 35.0687 35.1508 35.2329 35.3150 35.3971 35.4792 35.5613 35.6434 35.7255 35.8076 35.8897 35.9718 36.0539 36.1360 36.2181 36.3002 36.3823 36.4644 36.5465 36.6286 36.7107 36.7928 36.8749 36.9570 37.0391 37.1212 37.2033 37.2854 37.3675 37.4496 37.5317 37.6138 37.6959 37.7780 37.8601 37.9422 38.0243 38.1064 38.1885 38.2706 38.3527 38.4348 38.5169 38.5990 38.6811 38.7632 38.8453 38.9274 39.0095 39.0916 39.1737 39.2558 39.3379 39.4200 39.5021 39.5842 39.6663 39.7484 39.8305 39.9126 39.9947 40.0768 40.1589 40.2410 40.3231 40.4052 40.4873 40.5694 40.6515 40.7336 40.8157 40.8978 40.9799 41.0620 41.1441 41.2262 41.3083 41.3904 41.4725 41.5546 41.6367 41.7188 41.8009 41.8830 41.9651 42.0472 42.1293 42.2114 42.2935 42.3756 42.4577 42.5398 42.6219 42.7040 42.7861 42.8682 42.9503 43.0324 43.1145 43.1966 43.2787 43.3608 43.4429 43.5250 43.6071 43.6892 43.7713 43.8534 43.9355 44.0176 44.0997 44.1818 44.2639 44.3460 44.4281 44.5102 44.5923 44.6744 44.7565 44.8386 44.9207 45.0028 45.0849 45.1670 45.2491 45.3312 45.4133 45.4954 45.5775 45.6596 45.7417 45.8238 45.9059 45.9880 46.0701 46.1522 46.2343 46.3164 46.3985 46.4806 46.5627 46.6448 46.7269 46.8090 46.8911 46.9732 47.0553 47.1374 47.2195 47.3016 47.3837 47.4658 47.5479 47.6300 47.7121 47.7942 47.8763 47.9584 48.0405 48.1226 48.2047 48.2868 48.3689 48.4510 48.5331 48.6152 48.6973 48.7794 48.8615 48.9436 49.0257 49.1078 49.1899 49.2720 49.3541 49.4362 49.5183 49.6004 49.6825 49.7646 49.8467 49.9288 50.0109 50.0930 50.1751 50.2572 50.3393 50.4214 50.5035 50.5856 50.6677 50.7498 50.8319 50.9140 50.9961 51.0782 51.1603 51.2424 51.3245 51.4066 51.4887 51.5708 51.6529 51.7350 51.8171 51.8992 51.9813 52.0634 52.1455 52.2276 52.3097 52.3918 52.4739 52.5560 52.6381 52.7202 52.8023 52.8844 52.9665 53.0486 53.1307 53.2128 53.2949 53.3770 53.4591 53.5412 53.6233 53.7054 53.7875 53.8696 53.9517 54.0338 54.1159 54.1980 54.2801 54.3622 54.4443 54.5264 54.6085 54.6906 54.7727 54.8548 54.9369 55.0190 55.1011 55.1832 55.2653 55.3474 55.4295 55.5116 55.5937 55.6758 55.7579 55.8400 55.9221 56.0042 56.0863 56.1684 56.2505 56.3326 56.4147 56.4968 56.5789 56.6610 56.7431 56.8252 56.9073 56.9894 57.0715 57.1536 57.2357 57.3178 57.3999 57.4820 57.5641 57.6462 57.7283 57.8104 57.8925 57.9746 58.0567 58.1388 58.2209 58.3030 58.3851 58.4672 58.5493 58.6314 58.7135 58.7956 58.8777 58.9598 59.0419 59.1240 59.2061 59.2882 59.3703 59.4524 59.5345 59.6166 59.6987 59.7808 59.8629 59.9450 60.0271 60.1092 60.1913 60.2734 60.3555 60.4376 60.5197 60.6018 60.6839 60.7660 60.8481 60.9302 61.0123 61.0944 61.1765 61.2586 61.3407 61.4228 61.5049 61.5870 61.6691 61.7512 61.8333 61.9154 62.0000
```


We now define a function that computes the value of the function at any mesh point. You will find this function at the end of this script, and we copy it here for clarity

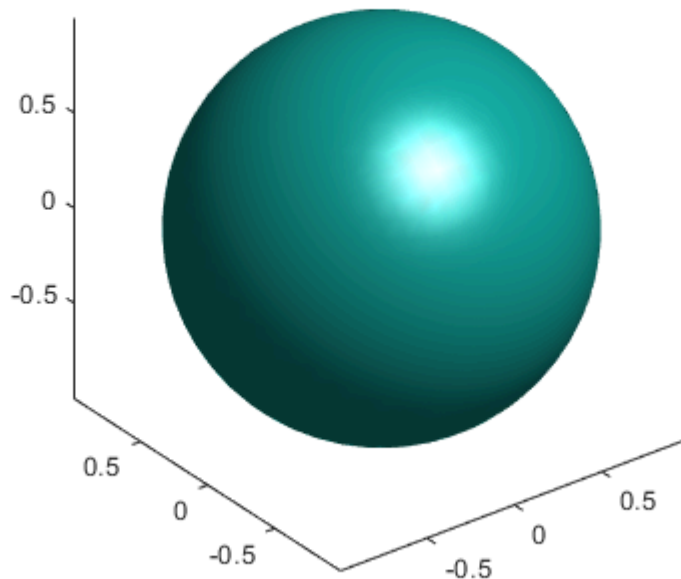
```
function res = sphere(x,y,z)
```

```
res = x.^2 + y.^2 + z.^2 - 1
```

```
end
```

Notice that we use the `.^` operator since the input arguments to this function can and will be matrices. Now we can call this function and visualize the surface

```
V = sphere(X,Y,Z);  
isosurface(X,Y,Z,V,0)  
axis('equal')
```



Generalization

There are lots of implicit surfaces, but a particularly important group is the quadratic (or quadric) surfaces, defined by the equation

$$Ax^2 + By^2 + Cz^2 + Dyz + Ezx + Fxy + Gx + Hy + Iz + J = 0$$

where A, B, C, D, E, F, G, H, I, and J are all arbitrary constants, some of which may be zero.

Exercise 1: Consider the surface defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$

- Write a function that accepts x, y, z, a, b, and c as input and returns the value of the expression on the left hand side.
- Visualize the surface for some different values of a, b, and c.
- What features of the surface do a, b, and c control?

```
x=linspace(-10,10,100);
y=linspace(-10,10,100);
z=linspace(-10,10,100);
[X,Y,Z] = meshgrid(x,y,z);
```

```
a=2
```

```
a = 2
```

```
V=myfunc(X,Y,Z,a,1,1);
```

```
res =
res(:,:,1) =
```

224.0000	223.0001	222.0206	221.0615	220.1228	219.2046	218.3067	217.4292	216.5722	215.7355	214.9193	214.
220.0004	219.0005	218.0210	217.0619	216.1233	215.2050	214.3071	213.4297	212.5726	211.7359	210.9197	210.
216.0824	215.0825	214.1031	213.1440	212.2053	211.2870	210.3891	209.5117	208.6546	207.8180	207.0017	206.
212.2461	211.2462	210.2667	209.3076	208.3689	207.4507	206.5528	205.6753	204.8183	203.9816	203.1654	202.
208.4914	207.4915	206.5120	205.5529	204.6142	203.6959	202.7981	201.9206	201.0636	200.2269	199.4107	198.
204.8183	203.8184	202.8389	201.8798	200.9411	200.0229	199.1250	198.2475	197.3905	196.5538	195.7376	194.
201.2268	200.2269	199.2474	198.2883	197.3497	196.4314	195.5335	194.6561	193.7990	192.9624	192.1461	191.
197.7170	196.7171	195.7376	194.7785	193.8398	192.9215	192.0237	191.1462	190.2892	189.4525	188.6363	187.
194.2887	193.2888	192.3094	191.3503	190.4116	189.4933	188.5954	187.7180	186.8609	186.0243	185.2080	184.
190.9421	189.9423	188.9628	188.0037	187.0650	186.1467	185.2489	184.3714	183.5143	182.6777	181.8614	181.
187.6772	186.6773	185.6978	184.7387	183.8000	182.8817	181.9839	181.1064	180.2494	179.4127	178.5965	177.
184.4938	183.4939	182.5144	181.5554	180.6167	179.6984	178.8005	177.9231	177.0660	176.2294	175.4131	174.
181.3921	180.3922	179.4127	178.4536	177.5149	176.5967	175.6988	174.8213	173.9643	173.1276	172.3114	171.
178.3720	177.3721	176.3926	175.4335	174.4948	173.5766	172.6787	171.8012	170.9442	170.1075	169.2913	168.
175.4335	174.4336	173.4541	172.4951	171.5564	170.6381	169.7402	168.8628	168.0057	167.1691	166.3528	165.
172.5767	171.5768	170.5973	169.6382	168.6995	167.7812	166.8834	166.0059	165.1489	164.3122	163.4960	162.
169.8014	168.8016	167.8221	166.8630	165.9243	165.0060	164.1082	163.2307	162.3736	161.5370	160.7207	159.
167.1078	166.1079	165.1285	164.1694	163.2307	162.3124	161.4145	160.5371	159.6800	158.8434	158.0271	157.
164.4959	163.4960	162.5165	161.5574	160.6187	159.7004	158.8026	157.9251	157.0681	156.2314	155.4152	154.
161.9655	160.9656	159.9861	159.0270	158.0884	157.1701	156.2722	155.3948	154.5377	153.7011	152.8848	152.
159.5168	158.5169	157.5374	156.5783	155.6396	154.7214	153.8235	152.9460	152.0890	151.2523	150.4361	149.
157.1497	156.1498	155.1703	154.2112	153.2725	152.3542	151.4564	150.5789	149.7219	148.8852	148.0690	147.
154.8642	153.8643	152.8848	151.9257	150.9870	150.0688	149.1709	148.2934	147.4364	146.5997	145.7835	144.
152.6603	151.6604	150.6810	149.7219	148.7832	147.8649	146.9670	146.0896	145.2325	144.3959	143.5796	142.
150.5381	149.5382	148.5587	147.5996	146.6610	145.7427	144.8448	143.9674	143.1103	142.2736	141.4574	140.
148.4975	147.4976	146.5181	145.5590	144.6203	143.7021	142.8042	141.9267	141.0697	140.2330	139.4168	138.
146.5385	145.5386	144.5591	143.6000	142.6614	141.7431	140.8452	139.9678	139.1107	138.2741	137.4578	136.
144.6612	143.6613	142.6818	141.7227	140.7840	139.8657	138.9679	138.0904	137.2333	136.3967	135.5805	134.
142.8654	141.8655	140.8860	139.9269	138.9883	138.0700	137.1721	136.2947	135.4376	134.6010	133.7847	132.
141.1513	140.1514	139.1719	138.2128	137.2742	136.3559	135.4580	134.5806	133.7235	132.8868	132.0706	131.
139.5188	138.5189	137.5394	136.5803	135.6417	134.7234	133.8255	132.9481	132.0910	131.2544	130.4381	129.
137.9680	136.9681	135.9886	135.0295	134.0908	133.1725	132.2747	131.3972	130.5401	129.7035	128.8873	128.
136.4987	135.4988	134.5193	133.5602	132.6216	131.7033	130.8054	129.9280	129.0709	128.2343	127.4180	126.
135.1111	134.1112	133.1317	132.1726	131.2340	130.3157	129.4178	128.5404	127.6833	126.8466	126.0304	125.
133.8051	132.8052	131.8257	130.8666	129.9280	129.0097	128.1118	127.2344	126.3773	125.5407	124.7244	123.
132.5808	131.5809	130.6014	129.6423	128.7036	127.7853	126.8875	126.0100	125.1529	124.3163	123.5001	122.
131.4380	130.4381	129.4586	128.4995	127.5609	126.6426	125.7447	124.8673	124.0102	123.1736	122.3573	121.
130.3769	129.3770	128.3975	127.4384	126.4997	125.5815	124.6836	123.8061	122.9491	122.1124	121.2962	120.
129.3974	128.3975	127.4180	126.4589	125.5203	124.6020	123.7041	122.8267	121.9696	121.1329	120.3167	119.
128.4995	127.4996	126.5202	125.5611	124.6224	123.7041	122.8062	121.9288	121.0717	120.2351	119.4188	118.
127.6833	126.6834	125.7039	124.7448	123.8061	122.8879	121.9900	121.1125	120.2555	119.4188	118.6026	117.

126.9487	125.9488	124.9693	124.0102	123.0715	122.1532	121.2554	120.3779	119.5209	118.6842	117.8680	117.0517
126.2957	125.2958	124.3163	123.3572	122.4185	121.5003	120.6024	119.7249	118.8679	118.0312	117.2150	116.4183
125.7243	124.7244	123.7449	122.7858	121.8472	120.9289	120.0310	119.1536	118.2965	117.4599	116.6436	115.8467
125.2346	124.2347	123.2552	122.2961	121.3574	120.4391	119.5413	118.6638	117.8068	116.9701	116.1539	115.3570
124.8264	123.8265	122.8471	121.8880	120.9493	120.0310	119.1331	118.2557	117.3986	116.5620	115.7457	114.9488
124.4999	123.5001	122.5206	121.5615	120.6228	119.7045	118.8067	117.9292	117.0721	116.2355	115.4192	114.6223
124.2551	123.2552	122.2757	121.3166	120.3779	119.4596	118.5618	117.6843	116.8273	115.9906	115.1744	114.3775
124.0918	123.0919	122.1124	121.1534	120.2147	119.2964	118.3985	117.5211	116.6640	115.8274	115.0111	114.2142
124.0102	123.0103	122.0308	121.0717	120.1330	119.2148	118.3169	117.4394	116.5824	115.7457	114.9295	114.1483
124.0918	123.0919	122.1124	121.1534	120.2147	119.2964	118.3985	117.5211	116.6640	115.8274	115.0111	114.2142
124.2551	123.2552	122.2757	121.3166	120.3779	119.4596	118.5618	117.6843	116.8273	115.9906	115.1744	114.3775
124.4999	123.5001	122.5206	121.5615	120.6228	119.7045	118.8067	117.9292	117.0721	116.2355	115.4192	114.6223
124.8264	123.8265	122.8471	121.8880	120.9493	120.0310	119.1331	118.2557	117.3986	116.5620	115.7457	114.9488
125.2346	124.2347	123.2552	122.2961	121.3574	120.4391	119.5413	118.6638	117.8068	116.9701	116.1539	115.3570
125.7243	124.7244	123.7449	122.7858	121.8472	120.9289	120.0310	119.1536	118.2965	117.4599	116.6436	115.8467
126.2957	125.2958	124.3163	123.3572	122.4185	121.5003	120.6024	119.7249	118.8679	118.0312	117.2150	116.4183
126.9487	125.9488	124.9693	124.0102	123.0715	122.1532	121.2554	120.3779	119.5209	118.6842	117.8680	117.0517
⋮											

```
isosurface(X,Y,Z,V,0);
axis('equal')
```

b=2

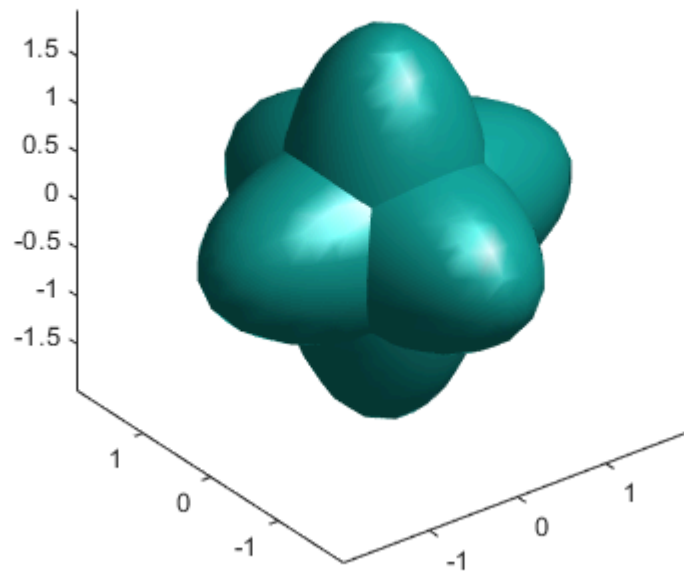
b = 2

```
V=myfunc1(X,Y,Z,1,b,1);
isosurface(X,Y,Z,V,0);
axis equal
```

c=2

c = 2

```
V=myfunc1(X,Y,Z,1,1,c);
isosurface(X,Y,Z,V,0);
axis equal
```



Function definitions

```
function res = sphere(x,y,z)
    res = x.^2 + y.^2 + z.^2 - 1;
end
function res=myfunc1(x,y,z,a,b,c)
    res=x.^2/a^2+y.^2/b^2+z.^2/c^2-1;
end
function res = myfunc(x,y,z,a,b,c)
    res = x.^2/a.^2 + y.^2/b.^2 + z.^2/c.^2 - 1
end
```